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The large-N reduction in matrix quantum mechanics — a bridge between BFSS and IKKT —

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Abstract: The large-N reduction is an equivalence between large-N gauge theories and matrix models discovered by Eguchi and Kawai in the early 80s. In particular the *continuum* version of the quenched Eguchi-Kawai (cQEK) model may be useful in studying supersymmetric and/or chiral gauge theories nonperturbatively. Unlike the original *lattice* version, however, the equivalence for the cQEK model is highly nontrivial. We study this model in the case of matrix quantum mechanics, i.e., the large-N gauge theory in one dimension. In the bosonic case we provide a direct confirmation of the equivalence by Monte Carlo simulations, and discuss an explanation based on the Schwinger-Dyson equations. In the supersymmetric case we argue that the equivalence holds as well possibly with some care when one takes the large-N limit. This equivalence provides an explicit relation between the BFSS model and the IKKT model, which are conjectured nonperturbative formulations of M-theory and type IIB superstring theory, respectively. In particular we conjecture that the SO(9) symmetry of the BFSS model is spontaneously broken if an analogous phenomenon occurs in the IKKT model as suggested by recent works.

Keywords: Matrix Models, Nonperturbative Effects.

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Contents

1. Introduction

The large-N reduction [1] states that large-N gauge theories are equivalent to matrix models obtained by dimensionally reducing those theories to a point. Recent developments in the exact calculation of the low-energy superpotential in $\mathcal{N} = 1$ supersymmetric gauge theories [2] may also be regarded as a particular application of the large-N reduction [3]. The ultimate aim of our work is to apply the original idea to supersymmetric gauge theories, but in a more conventional way. Our approach would be complementary to the lattice approach, in which there are new developments based on the idea of deconstruction [4, 5] and some techniques borrowed from the topological field theory [6]. We are going to use a large-N reduced model obtained from the *continuum* theory thereby avoiding the breaking of supersymmetry from the outset.

In ref. [1] it was conjectured that the U(N) gauge theory on an infinite lattice is equivalent in the large-N limit to a one-site model called the Eguchi-Kawai (EK) model. This conjecture was based on the observation that the Schwinger-Dyson equations of the two theories coincide under the assumption that the $\mathrm{U}(1)^d$ symmetry $U_\mu \to e^{i\alpha_\mu} U_\mu$ of the EK model remains unbroken. The $\mathrm{U}(1)^d$ symmetry, however, turned out to be spontaneously broken in the weak coupling region [7, 8], and the quenched EK (QEK) model was proposed as a remedy [7]. The "quenched" eigenvalues of U_μ were interpreted later as lattice momenta [9], and this interpretation enabled an extension to non-gauge theories. The equivalence for the QEK model was confirmed both perturbatively [10] and nonperturbatively [7, 11].

The *continuum* version of the QEK (cQEK) model has been proposed in ref. [12], and its potential usefulness in studying supersymmetric and/or chiral gauge theories has been

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emphasized. Unlike the *lattice* version, however, the EK equivalence for the cQEK model has not been checked directly so far, and it is actually highly nontrivial as we will see.

The twisted EK model [13], which was proposed as an alternative way to cure the problem of the original EK model, has been interpreted recently as field theories on a noncommutative geometry [14]. However, this model is not useful for our purpose since the continuum version can be defined only in the strict $N = \infty$ limit [15]. As another recent development, it has been found that the breaking of the $\mathrm{U}(1)^d$ symmetry can be avoided by keeping the size of the lattice sufficiently large, and that the critical size in physical units becomes finite in the continuum limit [16].

In this letter we consider the cQEK model for matrix quantum mechanics, i.e., the large-N gauge theory in one dimension as a first step, and discuss its EK equivalence. In the bosonic case we provide a direct confirmation of the equivalence by Monte Carlo simulations, and discuss an explanation based on the Schwinger-Dyson equations.

In the supersymmetric case we argue that the equivalence holds as well possibly with some care when one takes the large-N limit. As a particular application, we consider the BFSS Matrix Theory [17] proposed as a nonperturbative definition of M-theory [18]. The same model appeared earlier in ref. [19], and it may also be interpreted as a low-energy effective theory of D0-branes in type IIA superstring theory [20]. (In the cases where fermions are absent, interesting results are already obtained by a direct lattice approach [21]. See ref. [5] for a lattice formulation in the supersymmetric case.)

In fact the cQEK model for the BFSS model is analogous to the IKKT model [22] proposed as a nonperturbative formulation of type IIB superstring theory. Based on this observation, we discuss an explicit relation between the BFSS model and the IKKT model, which may be used to translate results in one model to the other. In particular we conjecture that the SO(9) symmetry of the BFSS model is spontaneously broken if an analogous phenomenon occurs in the IKKT model as suggested by recent works [23].

2. The models

We study a one-dimensional $\mathrm{U}(N)$ gauge theory, or matrix quantum mechanics (QM), with adjoint scalars $X_i(t)$ $(i = 1, \cdots, d-1)$, which are $N \times N$ hermitean matrices. The action is given by

$$S_{\mathrm{mQM}} = \frac{1}{g^2} \int dt \ \mathrm{tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\} , \tag{2.1}$$

where $DX_i = \frac{\partial}{\partial t} X_i - i\,[A, X_i]$ represents the covariant derivative with A being the one-dimensional gauge field. The model can be obtained formally by the dimensional reduction of d-dimensional Yang-Mills theory to one dimension. In what follows we fix the gauge to $A(t) = 0$. This model is UV finite and hence it does not require any UV regularization. As a result, the parameter g in (2.1) can always be scaled out by an appropriate rescaling of the matrices and the time coordinate t. We take $g = \frac{1}{\sqrt{N}}$, which turns out to be convenient when we discuss the large-N limit. Note that the model allows neither weak coupling nor strong coupling expansions, which usually provide a way to demonstrate the EK equivalence.

Applying the quenching prescription [9, 12] to the matrix QM (2.1), we obtain the corresponding cQEK model

$$S_{\text{cQEK},\Omega} = -N\,\epsilon\ \text{tr}\left(\frac{1}{2}\,[\Omega, X_i]^2 + \frac{1}{4}\,[X_i, X_j]^2\right), \tag{2.2}$$

where X_i $(1 \le i \le d-1)$ are $N \times N$ traceless hermitean matrices and $\Omega \equiv \text{diag}(\omega_1, \cdots, \omega_N)$ represents frequency variables to be integrated over the region $[-\frac{\Lambda}{2}, \frac{\Lambda}{2}]$, where $\Lambda \equiv \frac{2\pi}{\epsilon}$. (More precisely, the difference $\omega_i - \omega_j$ corresponds to the frequency.)

As U(N) invariant operators in the matrix QM (2.1), let us consider

$$\mathcal{O} = \frac{1}{N}\,\text{tr}\,X_{i_1}(t_1)\cdots X_{i_n}(t_n) \tag{2.3}$$

and the corresponding operator

$$\tilde{\mathcal{O}} = \frac{1}{N}\,\text{tr}\,\tilde{X}_{i_1}(t_1)\cdots \tilde{X}_{i_n}(t_n) \tag{2.4}$$

in the cQEK model (2.2), where $\tilde{X}_i(t) \equiv e^{i\Omega t}\,X_i\,e^{-i\Omega t}$. Then, according to the EK equivalence, we have the equality

$$\lim_{N\to\infty}\langle\mathcal{O}\rangle_{\text{mQM}} = \lim_{\Lambda\to\infty}\lim_{N\to\infty}\int d\Omega\,\langle\tilde{\mathcal{O}}\rangle_{\text{cQEK},\Omega}\,, \tag{2.5}$$

where $\langle\ \cdot\ \rangle_{\text{mQM}}$ and $\langle\ \cdot\ \rangle_{\text{cQEK},\Omega}$ represent the expectation values with respect to the actions (2.1) and (2.2), respectively, and the integration over Ω is defined by

$$\int d\Omega \equiv \prod_{i=1}^{N}\int_{-\frac{\Lambda}{2}}^{\frac{\Lambda}{2}}\frac{d\omega_i}{\Lambda}\,. \tag{2.6}$$

3. Monte Carlo simulation

We test the equivalence (2.5) directly by Monte Carlo simulation. In order to simulate the one-dimensional model (2.1), we discretize the time "t", and obtain the action

$$S_{\text{lat}} = Na\left\{\sum_{n=1}^{T-1}\frac{1}{2}\,\text{tr}\left(\frac{X_i(n+1)-X_i(n)}{a}\right)^2 - \sum_{n=1}^{T}\frac{1}{4}\,\text{tr}[X_i(n), X_j(n)]^2\right\}, \tag{3.1}$$

where a represents the lattice spacing. The $N \times N$ hermitean matrices $X_i(n)$ $(n = 1, \cdots, T)$ sit on each site, and we treat $X_i(1)$ and $X_i(T)$ as independent variables; i.e., we do *not* impose periodic boundary conditions.[1] In order to retrieve the original model (2.1), we have to take the $a \to 0$ and $\tau \equiv a\,T \to \infty$ limits, where the order of the limits should not matter. The system is invariant under $X_i(n) \mapsto X_i(n) + \beta_i\,\mathbf{1}$, and we fix the corresponding zero mode by imposing the condition $\sum_{n=1}^{T}\text{tr}\,X_i(n) = 0$.

[1] The motivation for this is to justify the gauge fixing $A(t) = 0$. If we imposed periodic boundary conditions, the gauge field $A(t)$ could not be gauged away completely, resulting in a slight complication, which we decided to avoid. The results in the thermodynamic limit $\tau \to \infty$, however, would be the same.

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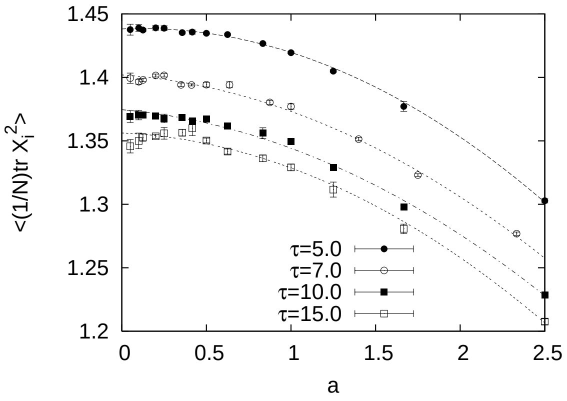

Figure 1: The observable $\langle \mathcal{O}_1 \rangle_{\text{lat}}$ for the lattice model (3.1) with $N = 16$ is plotted against a for $\tau = 5.0,\ 7.0,\ 10.0,\ 15.0$. The lines represent fits to $C_1 + C_2\, a + C_3\, a^2$.

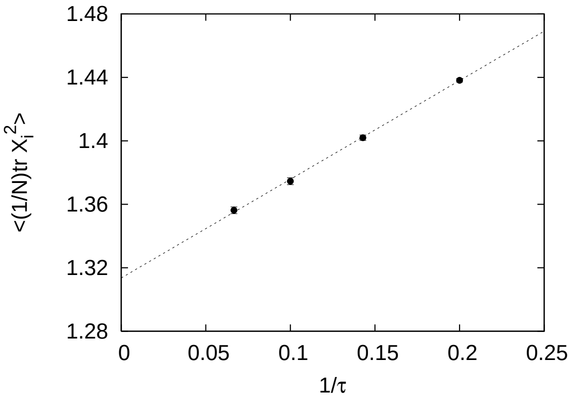

Figure 2: The observable $\langle \mathcal{O}_1 \rangle_{\text{lat}}$ for the lattice model (3.1) after taking the $a \to 0$ limit is plotted against $\frac{1}{\tau}$. The straight line represents a fit to $C_4 + C_5\, \tau^{-1}$.

Monte Carlo simulation of the models (3.1) and (2.2) can be performed by the heat bath algorithm [24]. For the cQEK model, we first take an average over the ensemble $\{X_i\}$ with the action (2.2) for each Ω, and then perform the averaging over Ω by using random numbers distributed uniformly within the region $[-\frac{\Lambda}{2}, \frac{\Lambda}{2}]$. Correspondingly there are two types of statistical errors. We estimate the total error by taking an average over the error of the first type and adding the one of the second type.

We focus on the $d = 4$ case in what follows. Let us first consider the observable[2] $\mathcal{O}_1 = \frac{1}{NT} \sum_{n=1}^{T} \operatorname{tr}(X_i(n)^2)$ for the lattice model (3.1). In figure 1 we plot the results for $\langle \mathcal{O}_1 \rangle_{\text{lat}}$ against a for $\tau = 5.0,\ 7.0,\ 10.0,\ 15.0$ at $N = 16$. We fit the results to $\langle \mathcal{O}_1 \rangle_{\text{lat}} = C_1 + C_2\, a + C_3\, a^2$, from which we obtain the continuum limit.[3] Figure 2 shows that the results obtained in the continuum limit for various τ can be nicely fitted to $\lim_{a \to 0} \langle \mathcal{O}_1 \rangle_{\text{lat}} = C_4 + C_5\, \tau^{-1}$, from which we obtain the $\tau \to \infty$ limit. We redo this analysis for $N = 20$, 32. The results can be fitted to $C_6 + C_7\, N^{-2}$ as one can see from figure 3. This large-N behavior is analogous to the one observed in the bosonic IKKT model [24]. Thus we can obtain the final result for the matrix QM (2.1).

Let us move on to the cQEK model (2.2). The corresponding observable is $\tilde{\mathcal{O}}_1 = \frac{1}{N} \operatorname{tr}(X_i)^2$. In figure 4 we plot the results $\int d\Omega\, \langle \tilde{\mathcal{O}}_1 \rangle_{\text{cQEK},\Omega}$ for $N = 32,\ 48,\ 64$ with $\epsilon = 0.2$, 0.25, 0.3, 0.5. For fixed ϵ the large-N behavior is given by $C_8 + C_9\, N^{-2}$. The coefficient C_9, however, diverges as $\frac{1}{\epsilon^2}$. (Therefore the order of the two limits on the r.h.s. of (2.5) cannot be inverted.) This can be understood if we recall that the density of ω_i in the region $[-\frac{\Lambda}{2}, \frac{\Lambda}{2}]$ is $\rho = \frac{N}{\Lambda} = \frac{N\epsilon}{2\pi}$. Since finite N effects at small ϵ come mainly from the finiteness of ρ, any power of N should be associated with the same power of ϵ. This motivated us

[2]Here we have taken an average over the lattice site n to increase the statistics, but one should obtain the same result for $\mathcal{O}_1(n) = \frac{1}{N} \operatorname{tr}(X_i(n)^2)$ in the $\tau \to \infty$ limit as far as the site n is well separated from the boundaries ($1 \ll n \ll T$).

[3]The coefficient C_2 of the linear term turns out to be tiny as one can see also from figure 1, but this is not the case for the other observables listed in table 1.

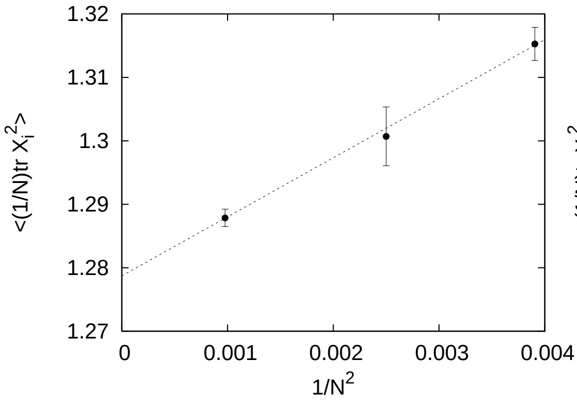

Figure 3: The observable $\langle \mathcal{O}_1 \rangle_{\text{lat}}$ for the lattice model (3.1) after taking the $a \to 0$ and $\tau \to \infty$ limits is plotted against $\frac{1}{N^2}$. The straight line represents a fit to $C_6 + C_7\, N^{-2}$.

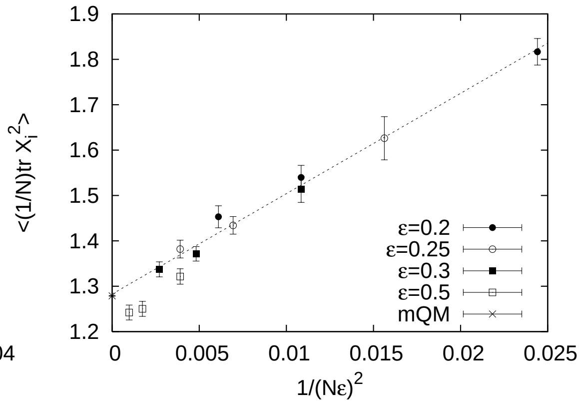

Figure 4: The observable $\int d\Omega \, \langle \tilde{\mathcal{O}}_1 \rangle_{\text{cQEK},\Omega}$ for the cQEK model (2.2) is plotted against $\frac{1}{(N\epsilon)^2}$ for $\epsilon = 0.2, 0.25, 0.3, 0.5$. The straight line represents a fit to $C_8 + C_{10}\,(N\epsilon)^{-2}$.

to plot the results against $\frac{1}{(N\epsilon)^2}$ in figure 4, where our data indeed lie on a single straight line for $\epsilon \le 0.3$. Fitting all the data for $\epsilon \le 0.3$, we obtain the final result for the cQEK model (2.2). The cross on the vertical axis (the error bar is almost invisible) represents the final result for the matrix QM (2.1), which is shown for comparison.

We repeat this analysis for other observables and summarize the results in table 1, where we find good agreement within error bars. We consider this as a compelling evidence for the EK equivalence (2.5).

observable	matrix QM	cQEK
$\frac{1}{N}\,\text{tr}(X_i)^2$	1.279(1)	1.28(1)
$\frac{1}{N}\,\text{tr}(X_i)^2(X_j)^2$	2.15(1)	2.11(5)
$\frac{1}{N}\,\text{tr}(X_i X_j)^2$	1.23(2)	1.27(4)

Table 1: Comparison of the results obtained for the matrix QM (2.1) and the cQEK model (2.2).

4. Schwinger-Dyson equations

Let us discuss a possible explanation for the equivalence (2.5) based on the Schwinger-Dyson (SD) equations [1, 12, 25].

In the matrix QM (2.1) we consider

$$\Big\langle \text{tr}(\lambda^a X_{i_1}(t_1) \cdots X_{i_n}(t_n)) \Big\rangle_{\text{mQM}} ,$$

where λ^a represents a generator of the $\text{U}(N)$ group. Changing the integration variables $X_i(t) \mapsto X_i(t) + \lambda^a \delta_{i,i_0} \delta(t - t_0)$, summing over the index a, using the identity $\sum_a (\lambda^a)_{ij} (\lambda^a)_{kl} = \delta_{il}\delta_{jk}$, and assuming the large-N factorization property $\langle \mathcal{O}\mathcal{O}' \rangle_{\text{mQM}} \simeq \langle \mathcal{O} \rangle_{\text{mQM}} \langle \mathcal{O}' \rangle_{\text{mQM}}$, we obtain a closed set of equations for the observables $\langle \mathcal{O} \rangle_{\text{mQM}}$, which is the SD equations.

In the cQEK model we consider

$$\int d\Omega \, \Big\langle \text{tr}(e^{i\Omega t_0} \lambda^a e^{-i\Omega t_0} \tilde{X}_{i_1}(t_1) \cdots \tilde{X}_{i_n}(t_n)) \Big\rangle_{\text{cQEK},\Omega} .$$

Changing the integration variables $X_i \mapsto X_i + \frac{1}{\epsilon}\lambda^a \delta_{i,i_0}$, we proceed similarly to the matrix QM case assuming in addition that the Ω-integration factorizes as well. In this way we obtain a set of equations, which are the same as the SD equations for the matrix QM with the identification (2.5) except for the terms $(s = 1, \cdots, n)$

$$\frac{1}{\epsilon}\,\delta_{i_0,i_s} \int d\Omega \left\langle \mathrm{tr}(e^{i\Omega(t_s-t_0)}\tilde{X}_{i_1}(t_1)\cdots\tilde{X}_{i_{s-1}}(t_{s-1}))\right\rangle_{\mathrm{cQEK},\Omega} \times \\ \times \int d\Omega \left\langle \mathrm{tr}(\tilde{X}_{i_{s+1}}(t_{s+1})\cdots\tilde{X}_{i_n}(t_n)e^{-i\Omega(t_s-t_0)})\right\rangle_{\mathrm{cQEK},\Omega},$$

whose counterpart exists in the matrix QM only when $t_s = t_0$. If we neglect the fact that the integration region of Ω is bounded by Λ, the system is invariant under the shift $\Omega \mapsto \Omega + \alpha\,\mathbf{1}$ except for the factors $e^{\pm i\Omega(t_s-t_0)}$, which make the two Ω-integrations vanish separately for $t_s \neq t_0$.

Since the model we are studying is UV finite, we may expect that the integrand for the Ω-integration is determined by the ω_i's which are clustered in a finite region. Then the integration over the position of the cluster gives a factor of $f(t_s - t_0) = \int_{-\Lambda/2}^{\Lambda/2} \frac{d\omega}{\Lambda} e^{i\omega(t_s-t_0)}$, and we obtain the coefficient $\frac{1}{\epsilon} f(t_s-t_0)^2 \to \delta(t_s-t_0)$, which agrees with the corresponding one in the matrix QM (2.1) in the $\epsilon \to 0$ limit. Assuming that the SD equations have a unique solution, we obtain the equality (2.5).

5. Relation to the bosonic IKKT model

The cQEK model (2.2) for the matrix QM has the action of the same form as the bosonic IKKT model [26]

$$S_{\mathrm{bIKKT}} = -\frac{1}{4}\,N\,\mathrm{tr}[A_\mu, A_\nu]^2 \tag{5.1}$$

if we make an identification

$$A_i \equiv \epsilon^{1/4}\,X_i \quad (\text{for} \;\; i \leq d-1) \tag{5.2}$$

$$A_d \equiv \epsilon^{1/4}\,\Omega\,. \tag{5.3}$$

The only difference is that A_d has to be quenched. In the bosonic IKKT model it is known that the SO(d) symmetry is not spontaneously broken and $\langle \frac{1}{N}\,\mathrm{tr}(A_\mu)^2\rangle_{\mathrm{bIKKT}} \sim \mathrm{O}(1)$ [24], which implies that the extent of the eigenvalue distribution is of O(1) in all directions. The quenching stretches the eigenvalue distribution of A_d to have an extent of $\mathrm{O}(\epsilon^{-3/4})$, and as a result the eigenvalue distribution of A_i ($i \leq d-1$) shrinks to $\mathrm{O}(\epsilon^{1/4})$ according to our result $\int d\Omega\,\left\langle \frac{1}{N}\,\mathrm{tr}(X_i)^2\right\rangle_{\mathrm{cQEK},\Omega} \sim \mathrm{O}(1)$. This implies that, in spite of their formal similarity, the cQEK model (2.2) and the bosonic IKKT model (5.1) probe totally different regions of the configuration space.

6. Supersymmetric case

The argument based on the SD equations formally goes through for the supersymmetric case as well. However, some subtlety may arise concerning the symmetry $\Omega \mapsto \Omega + \alpha\,\mathbf{1}$, which we would like to discuss below.

The $d = 4$ version of the IKKT model has been studied by Monte Carlo simulations [27], and it is known that the situation is qualitatively the same (no spontaneous breaking of the rotational symmetry and $\langle \frac{1}{N}\,\mathrm{tr}(A_\mu)^2\rangle \sim \mathrm{O}(1)$ [27]) as the bosonic IKKT model. Therefore we consider that the $d = 4$ version of the BFSS model can be studied by the corresponding cQEK model, which is, however, inequivalent to the $d = 4$ version of the IKKT model; i.e., quenching is necessary. It would be interesting to check these statements directly by Monte Carlo simulations.

In the IKKT model ($d = 10$) there are certain evidences that the eigenvalue distribution of A_μ collapses dynamically to a four-dimensional hypersurface and that the eigenvalue distribution extends to infinity in four directions at large N [23]. (See also refs. [28, 29] for related works on this issue.) Let us assume further that the distribution is uniform in the four extended directions, as suggested from the one-loop effective theory [28]. This is necessary for the translational invariance in our 4d space-time. When we quench the eigenvalues of A_{10}, the 4d hypersurface will arrange itself to have the required extent in the 10th direction. In this situation a vector in the 10th direction may have non-zero components in directions orthogonal to the 4d hypersurface, which implies that the system is not invariant under a shift in the 10th direction. We therefore suspect that the BFSS model is not equivalent to the corresponding cQEK model *as it stands*.

We may, however, add a "mass term" $\frac{1}{2}N\,m^2\int dt\ \mathrm{tr}\,X_i^2$ to the BFSS model, and consider the corresponding cQEK model, in which the mass term forces the 10th direction to be included in the 4d hypersurface. This guarantees the translational invariance in the 10th direction except at the boundary, which should be fine again due to the UV finiteness. Therefore the EK equivalence in this case is expected to hold at any finite m, and it should hold, too, even if we send m to zero eventually.

We speculate that actually quenching is not needed since the 10th direction, in which we do not have the mass term, will extend to infinity as $N \to \infty$ anyway. This implies that the IKKT model with a small mass term in 9 directions, which should be removed after taking the large-N limit, is equivalent (in the sense of Eguchi-Kawai) to the BFSS model. In particular if the IKKT model has 4 extended directions, the BFSS model should have 3 out of 9 transverse directions extended, thus breaking the SO(9) symmetry down to SO(3) spontaneously.

The equivalence between the BFSS model and the IKKT model we conjecture in this letter is somewhat reminiscent of the T-duality, which relates the BFSS model on a circle to the IKKT model with Taylor's compactifying condition [30]. Note, however, that our conjecture is different in that the two models can be *both* considered in the decompactified limit.

7. Concluding remarks

In this letter we have investigated the EK equivalence for the cQEK model in the case of matrix QM or one-dimensional large-N gauge theory. Whether it holds also in higher dimensions is an interesting open question. As is clear from the discussion on the SD equations, the question is highly nontrivial in the presence of UV divergences. In ref. [12]

it is speculated that the equivalence holds if the UV divergence is at most logarithmic. A possible first step in this direction would be to study the cQEK model for the 2d pure Yang-Mills theory, and see whether the known exact results can be reproduced.

Such a study would be relevant also to nonperturbative studies of Matrix String Theory [31], which describes type IIA superstrings in terms of two-dimensional $\mathcal{N}=8$ super Yang-Mills theory in the large-N limit. If the EK equivalence holds in this case, too, we may predict that the SO(8) symmetry of the theory is spontaneously broken down to SO(2), applying the same argument as in the previous section.[4] This implies that four-dimensional space-time appears nonperturbatively in type IIA superstring theory as suggested in type IIB superstring theory. In this way it might be possible to use the large-N reduction to demonstrate explicitly that different types of superstring theory actually describe the same unique vacuum if treated nonperturbatively.

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References

[1] T. Eguchi and H. Kawai, *Reduction of dynamical degrees of freedom in the large-N gauge theory*, *Phys. Rev. Lett.* **48** (1982) 1063.

[2] R. Dijkgraaf and C. Vafa, *A perturbative window into non-perturbative physics*, hep-th/0208048.

[3] H. Kawai, T. Kuroki and T. Morita, *Dijkgraaf-Vafa theory as large-N reduction*, *Nucl. Phys.* **B 664** (2003) 185 [hep-th/0303210].

[4] D.B. Kaplan, E. Katz and M. Unsal, *Supersymmetry on a spatial lattice*, *JHEP* **05** (2003) 037 [hep-lat/0206019];
A.G. Cohen, D.B. Kaplan, E. Katz and M. Unsal, *Supersymmetry on a euclidean spacetime lattice. I: a target theory with four supercharges*, *JHEP* **08** (2003) 024 [hep-lat/0302017]; *Supersymmetry on a euclidean spacetime lattice. II: target theories with eight supercharges*, *JHEP* **12** (2003) 031 [hep-lat/0307012].

[5] D.B. Kaplan and M. Unsal, *A euclidean lattice construction of supersymmetric Yang-Mills theories with sixteen supercharges*, hep-lat/0503039.

[6] S. Catterall, *Lattice supersymmetry and topological field theory*, *JHEP* **05** (2003) 038 [hep-lat/0301028]; *A geometrical approach to $N=2$ super Yang-Mills theory on the two dimensional lattice*, *JHEP* **11** (2004) 006 [hep-lat/0410052]; *Lattice formulation of $N=4$ super Yang-Mills theory*, *JHEP* **06** (2005) 027 [hep-lat/0503036];

[4] The two-dimensional space-time on which the gauge theory lives has an interpretation as the worldsheet of the type IIA superstrings, and hence one of the two directions should have a finite extent. However, this does not necessarily invalidate the equivalence since the one-point function of a single trace operator considered in the Eguchi-Kawai equivalence can be independent of the extent in the large-N limit.

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F. Sugino, *A lattice formulation of super Yang-Mills theories with exact supersymmetry*, *JHEP* **01** (2004) 015 [hep-lat/0311021]; *Super Yang-Mills theories on the two-dimensional lattice with exact supersymmetry*, *JHEP* **03** (2004) 067 [hep-lat/0401017]; *Various super Yang-Mills theories with exact supersymmetry on the lattice*, *JHEP* **01** (2005) 016 [hep-lat/0410035].

[7] G. Bhanot, U.M. Heller and H. Neuberger, *The quenched Eguchi-Kawai model*, *Phys. Lett.* **B 113** (1982) 47.

[8] M. Okawa, *Monte Carlo study of the Eguchi-Kawai model*, *Phys. Rev. Lett.* **49** (1982) 353.

[9] G. Parisi, *A simple expression for planar field theories*, *Phys. Lett.* **B 112** (1982) 463.

[10] S.R. Das and S.R. Wadia, *Translation invariance and a reduced model for summing planar diagrams in QCD*, *Phys. Lett.* **B 117** (1982) 228.

[11] M. Okawa, *Monte Carlo study of the quenched Eguchi-Kawai model*, *Phys. Rev. Lett.* **49** (1982) 705.

[12] D.J. Gross and Y. Kitazawa, *A quenched momentum prescription for large-N theories*, *Nucl. Phys.* **B 206** (1982) 440.

[13] A. Gonzalez-Arroyo and M. Okawa, *The twisted Eguchi-Kawai model: a reduced model for large-N lattice gauge theory*, *Phys. Rev.* **D 27** (1983) 2397.

[14] H. Aoki et al., *Noncommutative Yang-Mills in IIB matrix model*, *Nucl. Phys.* **B 565** (2000) 176 [hep-th/9908141];
J. Ambjørn, Y.M. Makeenko, J. Nishimura and R.J. Szabo, *Finite N matrix models of noncommutative gauge theory*, *JHEP* **11** (1999) 029 [hep-th/9911041]; *Nonperturbative dynamics of noncommutative gauge theory*, *Phys. Lett.* **B 480** (2000) 399 [hep-th/0002158]; *Lattice gauge fields and discrete noncommutative Yang-Mills theory*, *JHEP* **05** (2000) 023 [hep-th/0004147].

[15] A. Gonzalez-Arroyo and C.P. Korthals Altes, *Reduced model for large-N continuum field theories*, *Phys. Lett.* **B 131** (1983) 396.

[16] R. Narayanan and H. Neuberger, *Large-N reduction in continuum*, *Phys. Rev. Lett.* **91** (2003) 081601 [hep-lat/0303023]; *Chiral symmetry breaking at large N_c*, *Nucl. Phys.* **B 696** (2004) 107 [hep-lat/0405025]; *The quark mass dependence of the pion mass at infinite N*, *Phys. Lett.* **B 616** (2005) 76 [hep-lat/0503033];
J. Kiskis, R. Narayanan and H. Neuberger, *Proposal for the numerical solution of planar QCD*, *Phys. Rev.* **D 66** (2002) 025019 [hep-lat/0203005]; *Does the crossover from perturbative to nonperturbative physics in QCD become a phase transition at infinite N?*, *Phys. Lett.* **B 574** (2003) 65 [hep-lat/0308033];
T.D. Cohen, *Large-$N(c)$ continuum reduction and the thermodynamics of QCD*, *Phys. Rev. Lett.* **93** (2004) 201601 [hep-ph/0407306].

[17] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[18] E. Witten, *String theory dynamics in various dimensions*, *Nucl. Phys.* **B 443** (1995) 85 [hep-th/9503124].

[19] M. Claudson and M.B. Halpern, *Supersymmetric ground state wave functions*, *Nucl. Phys.* **B 250** (1985) 689.

[20] E. Witten, *Bound states of strings and p-branes*, *Nucl. Phys.* **B 460** (1996) 335 [hep-th/9510135].

[21] R.A. Janik and J. Wosiek, *Towards the matrix model of M-theory on a lattice*, *Acta Phys. Polon.* **B32** (2001) 2143 [hep-th/0003121];
P. Bialas and J. Wosiek, *Towards the lattice study of M-theory. II*, *Nucl. Phys.* **106** *(Proc. Suppl.)* (2002) 968 [hep-lat/0111034];
O. Aharony, J. Marsano, S. Minwalla and T. Wiseman, *Black hole - black string phase transitions in thermal 1+1 dimensional supersymmetric Yang-Mills theory on a circle*, *Class. and Quant. Grav.* **21** (2004) 5169 [hep-th/0406210].

[22] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *A large-N reduced model as superstring*, *Nucl. Phys.* **B 498** (1997) 467 [hep-th/9612115].

[23] J. Nishimura and F. Sugino, *Dynamical generation of four-dimensional space-time in the IIB matrix model*, *JHEP* **05** (2002) 001 [hep-th/0111102];
H. Kawai, S. Kawamoto, T. Kuroki, T. Matsuo and S. Shinohara, *Mean field approximation of IIB matrix model and emergence of four-dimensional space-time*, *Nucl. Phys.* **B 647** (2002) 153 [hep-th/0204240];
H. Kawai, S. Kawamoto, T. Kuroki and S. Shinohara, *Improved perturbation theory and four-dimensional space-time in IIB matrix model*, *Prog. Theor. Phys.* **109** (2003) 115 [hep-th/0211272].

[24] T. Hotta, J. Nishimura and A. Tsuchiya, *Dynamical aspects of large-N reduced models*, *Nucl. Phys.* **B 545** (1999) 543 [hep-th/9811220].

[25] Y.M. Makeenko, *Methods of contemporary gauge theory*, Cambridge, U.K., 2002.

[26] W. Krauth and M. Staudacher, *Finite Yang-Mills integrals*, *Phys. Lett.* **B 435** (1998) 350 [hep-th/9804199].

[27] J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, *Large-N dynamics of dimensionally reduced 4D* $\mathrm{SU}(N)$ *super Yang-Mills theory*, *JHEP* **07** (2000) 013 [hep-th/0003208];
J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, F. Hofheinz and J. Nishimura, *On the spontaneous breakdown of Lorentz symmetry in matrix models of superstrings*, *Phys. Rev.* **D 65** (2002) 086001 [hep-th/0104260];
Z. Burda, B. Petersson and M. Wattenberg, *Semiclassical geometry of 4D reduced supersymmetric Yang-Mills integrals*, *JHEP* **03** (2005) 058 [hep-th/0503032].

[28] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, *Space-time structures from IIB matrix model*, *Prog. Theor. Phys.* **99** (1998) 713 [hep-th/9802085].

[29] J. Ambjørn, K.N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, *Monte Carlo studies of the IIB matrix model at large N*, *JHEP* **07** (2000) 011 [hep-th/0005147];
J. Nishimura and G. Vernizzi, *Spontaneous breakdown of Lorentz invariance in IIB matrix model*, *JHEP* **04** (2000) 015 [hep-th/0003223]; *Brane world from IIB matrices*, *Phys. Rev. Lett.* **85** (2000) 4664 [hep-th/0007022];
Z. Burda, B. Petersson and J. Tabaczek, *Geometry of reduced supersymmetric 4D Yang-Mills integrals*, *Nucl. Phys.* **B 602** (2001) 399 [hep-lat/0012001];
J. Nishimura, *Exactly solvable matrix models for the dynamical generation of space-time in superstring theory*, *Phys. Rev.* **D 65** (2002) 105012 [hep-th/0108070];

K.N. Anagnostopoulos and J. Nishimura, *New approach to the complex-action problem and its application to a nonperturbative study of superstring theory*, *Phys. Rev.* **D 66** (2002) 106008 [hep-th/0108041];
G. Vernizzi and J.F. Wheater, *Rotational symmetry breaking in multi-matrix models*, *Phys. Rev.* **D 66** (2002) 085024 [hep-th/0206226];
T. Imai, Y. Kitazawa, Y. Takayama and D. Tomino, *Effective actions of matrix models on homogeneous spaces*, *Nucl. Phys.* **B 679** (2004) 143 [hep-th/0307007];
J. Nishimura, *Lattice superstring and noncommutative geometry*, *Nucl. Phys.* **129** *(Proc. Suppl.)* (2004) 121 [hep-lat/0310019];
T. Imai and Y. Takayama, *Stability of fuzzy $S^2 \times S^2$ geometry in IIB matrix model*, *Nucl. Phys.* **B 686** (2004) 248 [hep-th/0312241];
J. Nishimura, T. Okubo and F. Sugino, *Gaussian expansion analysis of a matrix model with the spontaneous breakdown of rotational symmetry*, hep-th/0412194;
S. Bal, M. Hanada, H. Kawai and F. Kubo, *Fuzzy torus in matrix model*, hep-th/0412303;
H. Kaneko, Y. Kitazawa and D. Tomino, *Stability of fuzzy $S^2 \times S^2 \times S^2$ in IIB type matrix models*, *Nucl. Phys.* **B 725** (2005) 93 [hep-th/0506033].

[30] W. Taylor, *D-brane field theory on compact spaces*, *Phys. Lett.* **B 394** (1997) 283 [hep-th/9611042].

[31] L. Motl, *Proposals on nonperturbative superstring interactions*, hep-th/9701025;
R. Dijkgraaf, E. Verlinde and H. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030].

[32] D.J. Gross and E. Witten, *Possible third order phase transition in the large-N lattice gauge theory*, *Phys. Rev.* **D 21** (1980) 446.